SCATTERING OF ELECTROMAGNETIC WAVES BY PLASMA

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ABSTRACT. In this paper we have treated the case of reflection and refraction of plane electromagnetic wave by plasma in an applied magnetic field. The component of the magnetic field perpendicular to the plane of incidence in the reflected wave provides an easy means of analysing the physical features of the plasma.

INTRODUCTION

The physical characteristics of a homogeneous plasma can be determined in various ways. We propose a simple way which utilises the anisotropic behaviour of plasma under an applied magnetic field. The external magnetic field thus as if removes the degeneracy. The various micro-effects are however not taken separately but their net effect is considered macroscopically. Thus the whole of the uniform plasma behaves like an anisotropic medium under the externally applied magnetic field. The co-efficients of anisotropy are dependent on the physical characteristics of the plasma. Thus the experimental determination of the field components of scattered electromagnetic fields from such a plasma will go to determine some of its internal details.

THE PLANE WAVES

Consider a semiinfinite homogeneous plasma. We choose our co-ordinates in such a way that

$Z = 0$ is the plasma boundary,

$Z > 0$ is plasma region,

$Z < 0$ is vacuum.

Let us suppose that there is an imposed external homogeneous magnetic field H parallel to the Z-direction. Under these circumstances the plasma region may be approximated as an anisotropic medium of dielectric tensor ϵ where

$$\epsilon = \begin{bmatrix} \epsilon_1 & \epsilon^* & 0 \\ \epsilon & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix} . \quad \ldots (1)$$

We now consider the propagation of a plane electromagnetic wave towards the plasma region where it will suffer reflection and refraction. The case when the incident wave will have the magnetic field perpendicular to the plane of incidence may be taken to discuss the situation. Calling the incident plane to be the $x-z$ plane we have

$$\vec{H} \text{ incident} = \vec{B}_0 \, e^{i\omega\left(t - \frac{\vec{r}.\vec{n}_0}{c}\right)} \qquad \ldots (2)$$

$$\vec{H} \text{ reflected} = \vec{B}_1 \, e^{i\omega\left(t - \frac{\vec{r}.\vec{n}_1}{c}\right)} \qquad \ldots (3)$$

and

$$\vec{H}' \text{ refracted} = \vec{B}_2 \, e^{i\omega\left(t - \frac{\vec{r}.\vec{n}_2}{v_2}\right)} \qquad \ldots (4)$$

$$\vec{H}'' \text{ refracted} = \vec{B}_3 \, e^{i\omega\left(t - \frac{\vec{r}.\vec{n}_3}{v_3}\right)} \qquad \ldots (5)$$

where $\vec{n}_i = \sin\theta_i \cos\phi_i \, \vec{i} + \sin\theta_i \sin\phi_i \, \vec{j} + \cos\theta_i \, \vec{k}$. ($\vec{i}, \vec{j}, \vec{k}$ are the unit vectors) c and v are the velocities of wave propagation in vacuum and plasma.

FIELD COMPONENTS

For the incident wave putting $\vec{B}_0 = \vec{Bj}$ and $\theta_0 = \theta =$ the angle of incidence and $\phi_0 = 0$ by choice, we obviously find from the continuity conditions of the magnetic field

$$\theta_1 = \theta, \quad \phi_i = 0 \text{ and } \frac{\sin\theta}{c} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta_3}{v_3} \qquad \ldots (6)$$

also

$$B + B_1 = B_2 + B_3 \qquad \ldots (7)$$

$$C_1 \cos\theta = -C_2 \cos\theta_2 - C_3 \cos\theta_3 \qquad \ldots (8)$$

$$C_1 \sin\theta = C_2 \sin\theta_2 + C_3 \sin\theta_3 \qquad \ldots (9)$$

We have called B_i and C_i the components of $\vec{B}_i$ in the incident plane and perpendicular to that plane respectively.

The electric displacement vector $\vec{D}$ will have its normal component continuous because of (7). We have the electric field vector

$$\vec{E} = -\frac{c}{v} \, E^{-1}[n \times H] \qquad \ldots (10)$$

The continuity of tangential components of $\vec{E}$ leads to

$$B \cos \theta - B_1 \cos \theta = \frac{c}{v_2} (a_1 B_2 \cos \theta_2 + a^* C_2) + \frac{c}{v_3} (a_1 B_3 \cos \theta_3 + a^* C_3) \quad \dots \quad (11)$$

and

$$C_1 = \frac{c}{v_2} (a B_2 \cos \theta_2 + a_2 C_2) + \frac{c}{v_3} (a B_3 \cos \theta_3 + a_2 C_3) \quad \dots \quad (12)$$

Where

$$\epsilon^{-1} = \begin{bmatrix} a_1 & a^* & 0 \\ a & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$$

we have further from Maxwell's equations.

$$\frac{v_2^2}{c^2} C_2 = a B_2 \cos \theta_2 + a_2 C_2 \quad (13)$$

$$\frac{v_3^2}{c^2} C_3 = a B_3 \cos \theta_3 + a_2 C_3 \quad \dots \quad (14)$$

$$\frac{v_2^2}{c^2} B_2 = a_1 B_2 \cos^2\theta_2 + a^* C_2 \cos \theta_2 + a_3 B_2 \sin^2\theta_2 \quad \dots \quad (15)$$

$$\frac{v_3^2}{c^2} B_3 = a_1 B_3 \cos^2 \theta_3 + a^* C_3 \cos \theta_3 + a_3 B_3 \sin^2 \theta_3 \quad \dots \quad (16)$$

The six quantities B_i and C_i can be obtained from the seven equations (7), (8), (9), (11), (12), (13) and (14) in terms of the incident intensity B. Equations (13) and (15) lead to the same equation as equations (14) and (16), e.g.

$$\{1 + (a_1 - a_3) \sin^2\theta\} \frac{v^4}{c^4} - \{a_1 + a_2 + (a_1 a_2 - a_2 a_3 - a a^*) \sin^2\theta\} \frac{v^2}{c^2} + a_1 a_2 - a a^* = 0 \quad \dots (17)$$

which really determine the two velocities inside the plasma, v_2 and v_3 and these along with equation (6) give the angle of refractions θ_2 and θ_3. From (17) we obtain

$$\frac{v^2}{c^2} = [a_1 + a_2 + (a_1 a_2 - a_2 a_3 - a a^*) \sin^2\theta \pm \sqrt{(\{a_1 + a_2 + (a_1 a_2 - a_2 a_3 - a a^*) \sin^2 \theta\}^2 - 4(a_1 a_2 - a a^*)\{1 + (a_1 - a_3) \sin^2 \theta\})}]/2\{1 + (a_1 - a_3) \sin^2 \theta\} \quad \dots \quad (18)$$

From equations (7) – (14) we get

$$B_1 = \frac{C_1}{2 \sin (\theta_2 - \theta_3)} \left[\frac{1}{a} \left(\frac{v_2^2}{c^2} - a_2 \right) \sin(\theta + \theta_3) \left(\frac{1}{\cos \theta_2} - \tan \theta \frac{a_1}{\sin \theta_2} \right) \right.$$

$$-\frac{1}{a}\left(\frac{v_3^2}{c^2}-a_2\right)\sin(\theta+\theta_2)\left(\frac{1}{\cos\theta_3}-\tan\theta\,\frac{a_1}{\sin\theta_3}\right)$$

$$-a^*\tan\theta\left\{\frac{\sin(\theta+\theta_3)}{\sin\theta_2}-\frac{\sin(\theta+\theta_2)}{\sin\theta_3}\right\}\Bigg] \quad \ldots \quad (19)$$

and $$C_1=2B\sin(\theta_2-\theta_3)\Bigg/\Bigg[\frac{1}{a}\left(\frac{v_2^2}{c^2}-a_2\right)\sin(\theta+\theta_3)\left(\frac{1}{\cos\theta_2}+\tan\theta\,\frac{a_1}{\sin\theta_2}\right)$$

$$-\frac{1}{a}\left(\frac{v_3^2}{c^2}-a_2\right)\sin(\theta+\theta_2)\left(\frac{1}{\cos\theta_3}+\tan\theta\,\frac{a_1}{\sin\theta_3}\right)$$

$$+a^*\tan\theta\left\{\frac{\sin(\theta+\theta_3)}{\sin\theta_2}-\frac{\sin(\theta+\theta_2)}{\sin\theta_3}\right\}\Bigg] \quad \ldots \quad (20)$$

We perticularly focus our attention to the reflected wave, since it will be outside the plasma and can be easily measured.

When the anisotropy will be small as is usually the case in the plasma under consideration we may make first order approximations and equations (18), (19) and (20) give

$$v_2=c\left\{1-\frac{1}{2}\eta+\frac{1}{4}(\eta+\delta)\sin^2\theta+\frac{1}{4}X\right\} \quad \ldots \quad (21)$$

$$v_3=c\left\{1-\frac{1}{2}\eta+\frac{1}{4}(\eta+\delta)\sin^2\theta-\frac{1}{4}X\right\} \quad \ldots \quad (22)$$

$$B_1=B\left\{-\frac{1}{4}\eta\sec^2\theta+\frac{1}{2}(\eta+\delta)\tan^2\theta\sin^2\theta\right\} \quad \ldots \quad (23)$$

$$C_1=-B\frac{\epsilon}{\cos\theta} \quad \ldots \quad (24)$$

where

$$\epsilon=\begin{bmatrix}1+\eta & \epsilon^* & 0\\ \epsilon & 1+\eta & 0\\ 0 & 0 & 1-\delta\end{bmatrix}$$

η, δ, ϵ are first order small quantities and

$$X=\sqrt{\{(\eta+\delta)^2\sin^4\theta+4\epsilon\epsilon^*\cos^2\theta\}}$$

MEASUREMENTS

If the plasma be kept without any applied field, no anisotropy occurs and C_1 in (24) is necessary zero. Applying a magnetic field, sending a polarized wide beam and subsequently measuring C_1 with a tuned coil (measuring the induced voltage) one can verify the result (24) and also get $|\epsilon|$. The waves should not be near grazing incidence as then (24) will not hold. This $|\epsilon|$ is

$$\sum_k \frac{\Omega_k \omega_k}{\omega(\omega_k^2 - \omega^2)}$$

A plot of (25) or even the induced voltage in the coil against the frequency will have peaking at ω_k. Also the values of $(\omega^2 - \omega_k^2)|\epsilon|$ near ω_k's will give the values of Ω_k. In the strict sense however one can not take ϵ to be given by (24) at the poles rather one should use (20). The expressions of Ω_k and ω_k being[1]

$$\Omega_k^2 = n_k e^2{}_k / m_k \qquad \ldots \quad (26)$$

$$\omega_k = e_k B / m_k. \qquad \ldots \quad (27)$$

The determinations of these give the values of the ionized masses and the fraction ionized, the state of ionization.

REFERENCE

Astrom 1951, *Arkiv fur Physik*. **2**.